XAVERIAN QUINTESSENCE

(An International Multidisciplinary Subjects and Peer- Reviewed Journal)

Biannual Online Journal

ISSN: 3139-6917 (Online)
Volume- 01, Issue-02
July 2026

Title of Research Paper: A Robust Multi-Objective Transportation Model under Budgeted Service and Uncertainty

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Abstract

Transportation planning requires an appropriate allocation of goods available from supply of sources to destination points while considering both cost and service requirements. In transportation systems, unit transportation costs and operating conditions may vary, making decisions based only on nominal data less reliable. This study newly develops a robust multi-objective transportation model that examines the trade-off between transportation cost and service performance under uncertainty. The baseline model combines supply and demand requirements with two competing objectives: minimization of robust transportation cost and maximization of service performance. After normalization, the baseline model uses a weighted-sum formulation represents different decision-maker preferences. As a proposed extension; a service-adjusted uncertainty exposure measure $H(x)$ is introduced to jointly account for maximum cost deviation, transported quantity, and service deficiency. The new criterion is normalized and incorporated through an additional weight w_H, producing a three-criterion formulation. The numerical example demonstrates and analyses the baseline model results, while the proposed formulation provides an interpretable mathematical extension for incorporating service-adjusted uncertainty exposure.

Keywords: Transportation model, Multi-objective optimization, Robust optimization, Budgeted uncertainty, Transportation cost, Service performance, Service-adjusted uncertainty exposure, Cost–service trade-off.

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XAVERIAN QUINTESSENCE
(An International Multidisciplinary Subjects and Peer- Reviewed Journal)

Biannual Online Journal

ISSN: 3139-6917 (Online)
Volume- 01, Issue-02
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1. Introduction

Transportation plays a central role in supply-chain and distribution systems because goods must be allocated from available sources to required destinations. An important planning problem is to determine how much should be transported along each route while keeping transportation expenditure under control and maintaining an appropriate level of service. In transportation systems, however, costs, available supply, demand, and service-related conditions may change with time. However, a mathematical formulation that incorporates both multiple objectives and uncertainty can provide a more useful basis for transportation decisions.

Conventional transportation models generally seek a minimum-cost allocation subject to supply and demand requirements. Although such a solution can be economically attractive, it may not provide the desired level of service. Conversely, improving service performance can require the use of routes or allocations with higher transportation costs. This conflict between economic efficiency and service quality naturally leads to a multi-objective transportation formulation.

Uncertainty further complicates transportation planning. Variations in fuel prices, road conditions, travel times, market conditions, and operational requirements can cause actual transportation costs to differ from their nominal values. A plan obtained from nominal parameters may therefore perform differently when implemented. Robust optimization addresses this issue by allowing possible parameter deviations to be considered during decision-making. In particular, a budgeted uncertainty approach limits the overall extent of simultaneous deviations and provides a controllable compromise between protection against adverse conditions and excessive conservatism.

Although transportation optimization, multi-objective modelling, and robust optimization have each received considerable attention, there is still a need for transparent formulations that examine cost, service performance, and controlled uncertainty together. In particular, it is useful to study how the balance between transportation cost and service performance changes as the allowable level of uncertainty protection is varied. This innovative study therefore develops a robust multi-objective transportation framework based on budgeted uncertainty.

XAVERIAN QUINTESSENCE
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ISSN: 3139-6917 (Online)
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The main thrust of the proposed framework is to jointly model robust transportation cost and service performance while controlling the degree of uncertainty through a budget parameter. This mathematical model is designed to provide transportation decisions that reflect a balance between economic efficiency, service quality, and robustness. A real life example is demonstrated to formulate and examine the resulting cost–service and trade-off under different uncertain circumstances based on the desired budget.

2. Literature Review

Transportation optimization has been widely investigated using mathematical programming and related optimization techniques. Traditional transportation models mainly concentrate on allocating goods between supply sources and destinations, with transportation cost as a principal objective. Recent studies have extended transportation models to incorporate robustness, multiple objectives, uncertainty, environmental concerns, and service-related requirements.

Recent research has developed robust transportation formulations for different uncertainty settings. Guo et al. [1] proposed a collaborative optimization strategy for multiperiod procurement and multimodal transportation that considers procurement, transportation, transhipment, and storage costs and constructs robust optimization models for the uncertain environment. Ren et al. [2] developed a hybrid robust stochastic optimization model for multimodal transportation route selection with uncertainty in demand, transportation time, and carbon-trading price. Grunau et al. [3] discussed on a General Polyhedral Approximation framework for two-stage robust linear programs under budgeted uncertainty and considered its approximation for a Transportation Location Problem. Zhang et al. [4] formulated a robust multimodal route-planning problem integrating shared taxis and buses under uncertain travel time, with objectives of reducing shared-taxi travel distance and improving passenger satisfaction.

Multi-objective and uncertainty-aware transportation models have also been developed for different application settings. Manafi and Sayan [5] proposed a bi-objective mixed-integer linear model for resilient and responsive global supply-chain design with multimodal transportation, minimizing total operational cost while maximizing customer responsiveness through service

ISSN: 3139-6917 (Online)
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levels and fulfilment lead times. Du [6] developed a hybrid robust optimization model for multimodal transportation routes under demand uncertainty and optimized transportation, transhipment, and time-related costs. Liu et al. [7] formulated a robust multimodal transport-path optimization model under uncertain cargo demand from a low-carbon perspective, incorporating transportation, transhipment, and waiting costs. Sahoo et al. [8] proposed a green multi-objective restricted fixed-charge four-dimensional transportation model with route-safety and budget constraints using uncertain interval programming. Bera et al. [9] formulated a multi-objective sustainable waste-transportation model under two-fold uncertainty, minimizing transportation cost and carbon emissions while addressing public-health impacts. Kapoor et al. [10] proposed a Fermatean fuzzy programming approach for a nonlinear multi-objective transportation problem under uncertainty, using an alpha-beta cut transformation to obtain an interval-valued transportation problem.

Robust optimization provides a systematic framework for handling uncertain parameters in optimization models. Bertsimas and Sim [11] introduced the price-of-robustness framework, in which the uncertainty budget provides a controllable balance between protection against uncertainty and excessive conservatism. This concept is relevant to transportation models in which the overall level of simultaneous parameter deviations needs to be controlled.

Transportation network design under uncertainty has also been investigated using robust approaches. Ukkusuri et al. [12] studied robust transportation network design under demand uncertainty and considered the effect of uncertainty on transportation network decisions. Chen et al. [13] reviewed transportation network design problems under uncertainty and discussed developments related to robust, reliable, and multi-objective transportation network design. Yin and Lou [14] developed a robust optimization approach for transportation network design under demand uncertainty, incorporating uncertain travel demand within a robust optimization framework.

Multi-objective transportation models have also incorporated reliability under uncertain conditions. Maity et al. [15] studied a multi-objective transportation problem with cost reliability under an uncertain environment, demonstrating the importance of considering reliability along with transportation cost in uncertain transportation planning.

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The reviewed studies demonstrate substantial progress in robust, multi-objective, sustainable, fuzzy, and uncertainty-aware transportation optimization. However, the existing studies mainly address specific settings such as multimodal procurement and routing, uncertain route selection, transportation location, passenger route planning, resilient supply chains, low-carbon transport, green transportation, waste management, and fuzzy transportation. There remains scope for a comparatively transparent transportation allocation framework that explicitly examines the cost–service trade-off under a controllable budgeted uncertainty mechanism.

In addition, the reviewed literature does not explicitly formulate the service-adjusted uncertainty exposure measure proposed in this study, which jointly considers maximum cost deviation, transported quantity, and service deficiency at the route level. Therefore, the present study first establishes a baseline robust cost–service transportation model and then introduces a service-adjusted uncertainty exposure criterion as an additional objective. This provides an extended framework for simultaneously examining transportation cost, service performance, and uncertainty exposure.

Research gap and contribution: The reviewed literature does not explicitly formulate the service-adjusted uncertainty exposure measure used in this study, namely$H(x)$, which jointly links maximum cost deviation, transported quantity, and service deficiency at route level. The present study first establishes a baseline robust cost-service transportation model and then proposes the service adjusted uncertainty exposure measure $H(x)$, its normalized form, and the additional weight w_H in a three-criterion weighted-sum formulation.

In the basic formulation, $I\ and\ J$ denote the sets of supply sources and destinations; $i\ and\ j$ are their indices; x_{ij} is the transported quantity; a_i and b_j are available supply and required demand; C_{ij} is nominal unit transportation cost; R_{ij} is service performance; $\tilde{C}_{ij}$is maximum unit-cost deviation; and Γ is the uncertainty-budget parameter.

3. Basic Concepts

Consider a transportation system with m supply sources and n destinations. Let $I = \{1, 2, \dots, m\}$ and $J = \{1, 2, \dots, n\}$ denote the sets of sources and destinations, with $i \in I$ and

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$j \in J$ as the corresponding indices. The decision variable $x_i \geq 0$ denotes the quantity transported from source i to destination j.

Here, a_i is the available supply at source i, b is the required demand at destination j, C_i is the nominal unit transportation cost, R_i is the service-performance coefficient, $\tilde{c}_i$ is the maximum possible unit-cost deviation, and Γ is the uncertainty-budget parameter.

The transportation allocation satisfies $\sum_{j \in J} x_i \leq a_i \; for \; all \; i \in I \; and \; \sum_{i \in I} x_i \geq b \; for \; all \; j \in J$.
The nominal transportation cost and service performances are defined as:

$Z^c(x) = \sum_{i \in I} \sum_{j \in J} C_i x_i \; and \; Z^s(x) = \sum_{i \in I} \sum_{j \in J} R_i x_i$ respectively.

For uncertain transportation cost, ξ_i denotes the normalized deviation, with $0 \leq \xi_i \leq 1$. The budgeted uncertainty parameter Γ controls the aggregate simultaneous deviations; $\Gamma = 0$ represents no deviation, while larger Γ permits greater adverse deviation. The resulting worst-case transportation cost is denoted by $Z^{Rc}(x)$, and its linear robust counterpart is used for numerical optimization.

The worst- case robust transportation cost is defined as the nominal transportation cost plus the maximum admissible cost deviation under the budgeted uncertainty set.

$$Z_{raw}^{RC} = Z^c(x) + \max_{\xi} \sum_i \sum_j \bar{c}_i x_i \xi_i : \sum_i \sum_j \xi_i \leq \Gamma$$

The baseline model therefore considers two objectives: minimizing robust transportation cost and maximizing service performance. Since these objectives have different directions and scales, they are converted into minimization-oriented normalized criteria before applying the weighted-sum formulation.

$Z^{RC}(x) = [\, Z_{raw}^{RC} - Z_{min}^{RC}] \,/\, [Z_{max}^{RC} - Z_{min}^{RC}]$

$Z^{S}(x) = [\, Z_{max}^{S} - Z_{raw}^{S}\,(x)] \,/\, [Z_{max}^{S} - Z_{min}^{S}]$

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XAVERIAN QUINTESSENCE
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ISSN: 3139-6917 (Online)
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Here, $Z^{\mathrm{Rc}}(x)$ is the normalized robust transportation cost, while $Z^{\mathrm{s}}(x)$is the normalized service- deficiency criterion. A smaller value of each criterion is preferred.The normalized criteria are combined using the following weighted-sum formulation:

$$F_0(x) = \lambda Z^{\mathrm{Rc}}(x) + (1 - \lambda) Z^{\mathrm{s}}(x)$$

The baseline problem is to minimize $F_0(x)$ subject to the transportation constraints, where $0 \leq \lambda \leq 1$.

4.Methodology: Service-Adjusted Uncertainty Model

The baseline model considers two objectives: robust transportation cost and service performance. The proposed extension adds a third criterion that measures uncertainty exposure together with service deficiency. Define the normalized service score on route (i, j) by

$$r_{ij} = (R_{ij} - R_{\min})/(R_{\max} - R_{\min}).$$

Here, r_{ij} is the normalized service score,R_{ij} is the route service coefficient, and $R_{\min}$ and $R_{\max}$ are the minimum and maximum service coefficients over all routes.The corresponding service deficiency is defined as $d_{ij} = 1 - r_{ij}$. Here, $1 - r_{ij}$ represents the service deficiency on route (i, j).

$$H(x) = \sum_{i \in I} \sum_{j \in J} \bar{c}_{ij} x_{ij} d_{ij} = \sum_{i \in I} \sum_{j \in J} \bar{c}_{ij} x_{ij} (1 - r_{ij}).$$

Here,$\bar{c}_{ij}$ measures possible cost deviation, x_{ij} measures transported quantity, and $(1 - r_{ij})$ measures service deficiency. Thus $H(x)$ links uncertainty exposure with the quantity transported and the quality of service on each route.

Let X denotes the set of all feasible transportation allocations satisfying the supply and demand constraints. The minimum and maximum feasible values of $H(x)$are defined as

$H_{\min} = min_{x \in X} H(x)andH_{\max} = max_{x \in X} H(x)$

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Here, H_{min} represents the minimum achievable service-adjusteduncertainty exposure among all feasible allocations, whereas H_{max} represents the maximum achievable service-adjusted uncertainty among all feasible allocations.

$$\widehat{H}(x) = \frac{[H(x) - H_{min}]}{[H_{max} - H_{min}]}.$$

The normalization transforms the proposed exposure criterion to a dimensionless scale between 0 to 1, making it compatible with the normalized robust-cost and service – performance criteria in the weighted-sum formulation. A smaller value of $\widehat{H}(x)$ indicates lower service –adjusted uncertainty exposure relative to the feasible range.The three criteriaare combined using the following weighted sum formulation:

$$F(x) = w_c Z^{cR}(x) + w_s Z^{s}(x) + w_H \widehat{H}(x)$$

Here, $w_c$is the robust-cost weight, w_s is the service-performance weight, and w_H is the new uncertainty-exposure weight. The weights satisfy $w_c \geq 0, w_s \geq 0, w_H \geq 0$ and $w_c + w_s + w_H = 1$. When $w_H = 0$, the proposed three-criterion formulation reduces to the existing two-objective weighted-sum model. Thus, the proposed formulation is a direct extension of the baseline model.

5. Application

The numerical demonstration below uses the same numerical instance introduced in the baseline model. For illustration, a feasible allocation is selected and the proposed service-adjusted uncertainty exposure criterion is evaluated for this allocation. The baseline cost and service measures are then combined with the proposed criterion to demonstrate the extended formulation.

The numerical instance uses the cost matrix$C = \begin{bmatrix} 4 & 6 & 8 \\ 5 & 4 & 7 \\ 6 & 5 & 3 \end{bmatrix}$

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(An International Multidisciplinary Subjects and Peer- Reviewed Journal)

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The service-performance matrix is $R = \begin{bmatrix} 1 & 10 & 12 \\ 9 & 1 & 11 \\ 10 & 8 & 1 \end{bmatrix}$

The maximum unit-cost deviation is defined as $\bar{c}_{ij} = 0.20C_{ij}$

Hence, $\check{C} = \begin{bmatrix} 0.8 & 1.2 & 1.6 \\ 1 & 0.8 & 1.4 \\ 1.2 & 1 & 0.6 \end{bmatrix}$

For the service normalization, $R_{i} = 1$, $_{ax} = 12$.

Therefore, $r_{i} = \frac{R_{i} - R_{i}}{R_{ax} - R_{i}} = \frac{R_{i} - 1}{12 - 1} = \frac{R_{i} - 1}{11}$

The corresponding service-deficiency is defined by

$$d_{i} = (1 - r_{i}) = \begin{bmatrix} 1 & 0.181818 & 0 \\ 0.272727 & 1 & 0.090909 \\ 0.181818 & 0.363636 & 1 \end{bmatrix}$$

The route-level coefficient in the proposed exposure measure is $\bar{c}ij(1 - r_{i})$, resulting

$$\begin{bmatrix} 0.8 & 0.218182 & 0 \\ 0.272727 & 0.8 & 0.127273 \\ 0.218182 & 0.363636 & 0.6 \end{bmatrix}$$

For the numerical illustration, consider three supply sources with available supplies $a = (25,35,25)$

And three destinations with required demands $b = (30,30,25)$

For illustration, consider the following feasible allocation:

$$X = \begin{bmatrix} 0 & 14.29 & 10.71 \\ 22.76 & 0 & 12.24 \\ 7.24 & 15.71 & 2.04 \end{bmatrix}$$

Using this allocation, the service –adjusted uncertainty is calculated as

XAVERIAN QUINTESSENCE
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$$H(x) = \sum_{i\in I}\sum_{j\in J} \bar{c}_{ij}x_{ij}(1 - r_{ij}) = \begin{bmatrix} 0 & 3.1178 & 0 \\ 6.2073 & 0 & 1.5578 \\ 1.5796 & 5.7127 & 1.2240 \end{bmatrix}$$

Thus, $H(x) = 19.399273$

To determine the feasible range of the proposed exposure measure, H(x) is minimized and maximized over the feasible transportation set X. The minimum value is obtained at the feasible allocation.

$$X_{min} = \begin{bmatrix} 0 & 25 & 0 \\ 10 & 0 & 25 \\ 20 & 5 & 0 \end{bmatrix}$$

For this allocation, $H_{min} = 17.545455$

Similarly, maximizing H(x) over the feasible set gives the feasible allocation.

$$X_{max} = \begin{bmatrix} 25 & 0 & 0 \\ 5 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

For this allocation, $H_{max} = 60.363636$

Thus, the feasible range of the service –adjusted uncertainty exposure is

$$17.545455 \leq H(x) \leq 60.363636$$

The normalized service-adjusted uncertainty is

$$\widehat{H}(x) = \frac{[H(x) - H_{min}]}{[H_{max} - H_{min}]}$$

$$\widetilde{H}(x) = [19.399273 - 17.545455]/[60.363636 - 17.545455]$$

$\widetilde{H}(x) = 0.043296$

The normalized criteria are combined using the following weighted-sum formulation:

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$$F(x) = w_c Z^{cR}(x) + w_s Z^s(x) + w_H \tilde{H}(x)$$

For the baseline case Γ = 2 andλ =0.50, the robust transportation cost is 478.23, the nominal transportation cost is 437.69 and service performance is 635.00.

Now $Z^{cR}(x) = \frac{478.23-364}{585-364} = 0.516878$

And $Z^S(x) = \frac{855-635}{855-125} = 0.301370$

For a transparent numerical demonstration only, take$w_C = 0.40$, w_S = 0.40 and w_H = 0.20. These weights are illustrative.

$$F(x) = 0.40\ (0.516878) + 0.40\ (0.301370) + 0.20\ (0.043296) \approx 0.335958$$

The resulting value $F(x) = 0.335958$ represents the combined normalized objective value for the illustrative weighted scheme. A smaller value of $F(x)$ indicates a lower combined value of the three normalized criteria under the selected weighted schemes.

Γ	λ	Robust Cost	Nominal Cost	Service Performance
0	0.00	520.00	520.00	855.00
0	0.25	520.00	520.00	855.00
0	0.50	395.00	395.00	550.00
0	0.75	320.00	320.00	125.00
0	1.00	320.00	320.00	125.00
1	0.00	555.00	520.00	855.00
1	0.25	530.00	509.00	834.17
1	0.50	425.00	395.00	550.00
1	0.75	344.00	320.00	125.00

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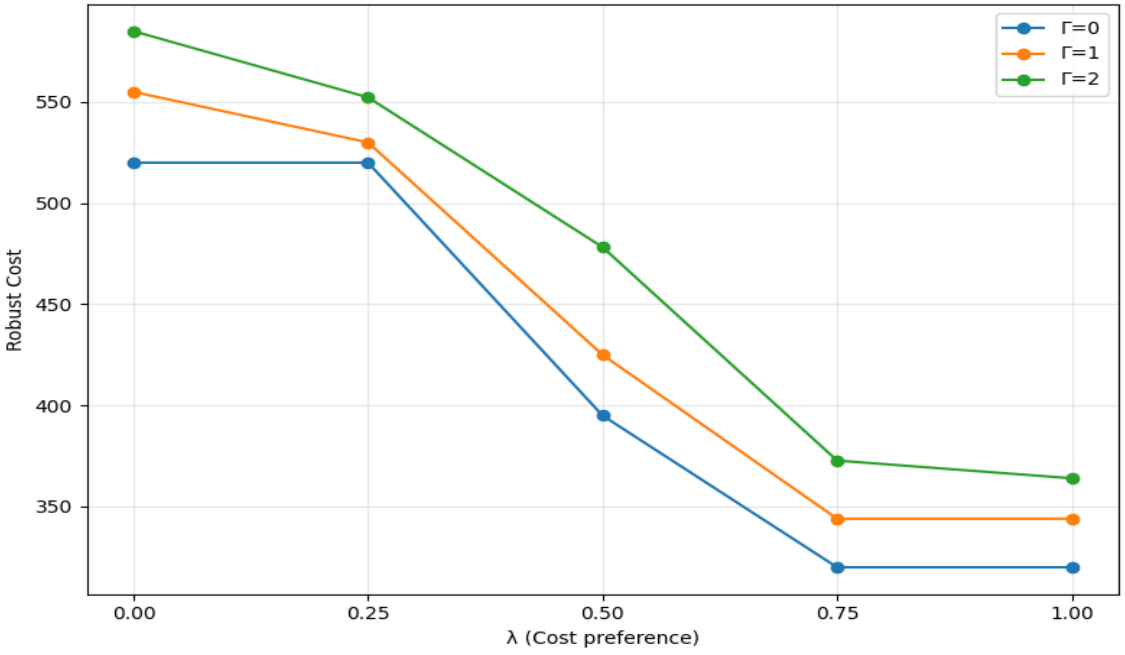
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ISSN: 3139-6917 (Online)
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1	1.00	344.00	320.00	125.00
2	0.00	585.00	520.00	855.00
2	0.25	552.27	510.91	836.82
2	0.50	478.23	437.69	635.00
2	0.75	372.75	338.75	231.25
2	1.00	363.00	320.00	125.00

Table 1. Baseline computational results for different uncertainty budgets and cost–service preferences

The table presents the baseline transportation-model results for each combination of λ and Γ. It reports robust transportation cost, nominal transportation cost, and service performance. These values are used as reference results for evaluating the proposed extension.



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XAVERIAN QUINTESSENCE
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Biannual Online Journal

ISSN: 3139-6917 (Online)
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Figure 1. Robust transportation cost under different cost–service preferences and uncertainty budgets: The graph presents robust transportation cost for the considered values of the cost–service preference parameter λ and uncertainty budget Γ. An increase in λ gives greater importance to cost minimization in the baseline weighted formulation, so the robust-cost values generally move toward lower-cost allocations. The different Γ values show the effect of protection against adverse cost deviations: a larger uncertainty budget allows more cost coefficients to deviate adversely and can therefore increase the worst-case robust transportation cost.

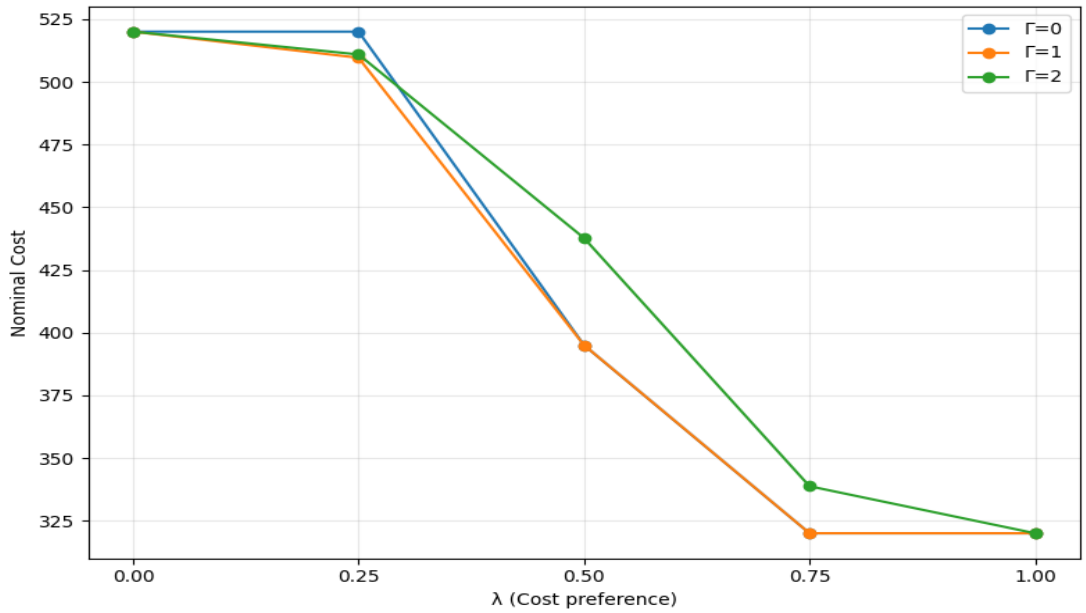


Figure 2. Nominal transportation cost under different cost–service preferences and uncertainty budgets: The graph presents nominal transportation cost for different values of the cost- service preference parameter λ and uncertainty budget Γ. As λ increases, greater importance is assigned to transportation-cost minimization in the baseline weighted formulation, and the nominal cost generally decreases. The comparison across different value

XAVERIAN QUINTESSENCE
(An International Multidisciplinary Subjects and Peer- Reviewed Journal)

Biannual Online Journal

ISSN: 3139-6917 (Online)
Volume- 01, Issue-02
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of Γ shows that the nominal cost is primarily determined by the selected cost-service preference, while the uncertainty budget affects the robust counterpart through protection against adverse cost deviations.

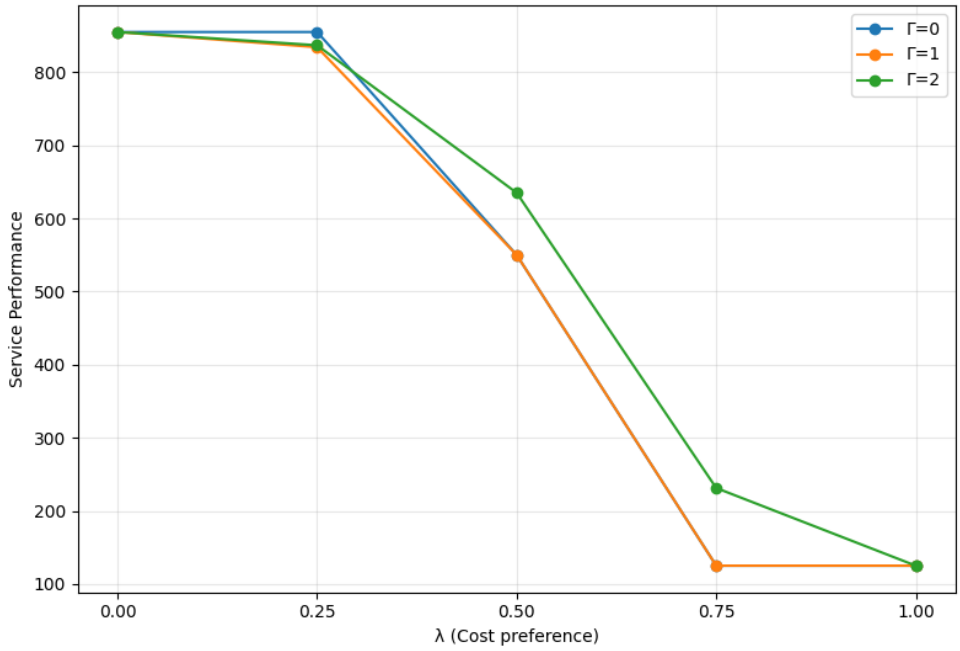


Figure 3. Service performance under different cost–service preferences and uncertainty budgets

The graph presents service performance for different values of the cost-service performance parameter λ and uncertainty budget Γ. Lower values of λ give greater importance to service performance, Whereas higher values give greater importance to transportation cost. Consequently, service performance generally decreases as λ increases. The different Γ values illustrate the effect of the uncertainty budget on the robust cost-service trade off.

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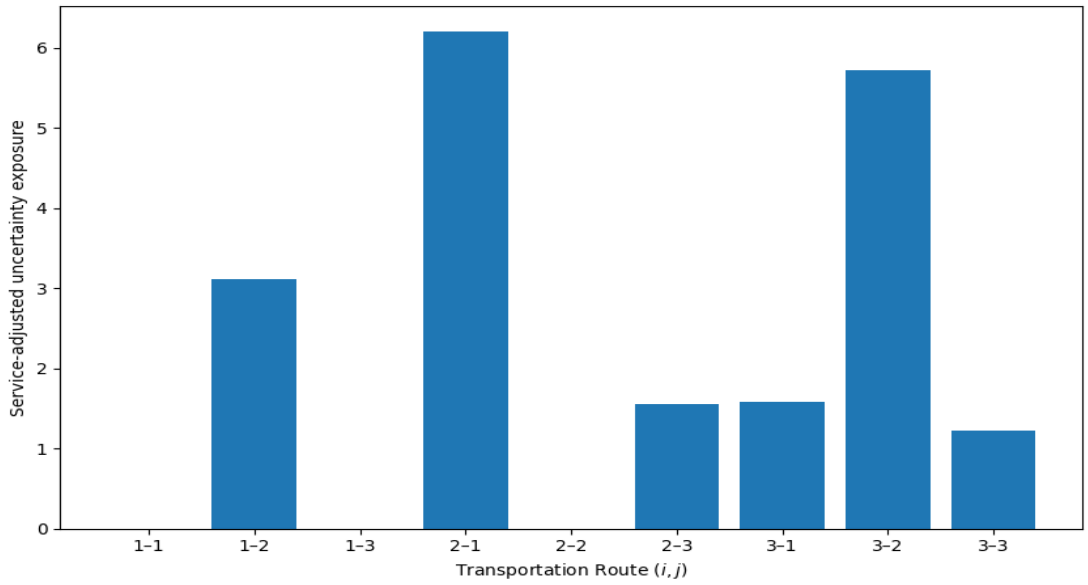


Figure 4. Service-adjusted uncertainty exposure for the numerical illustration: The figure illustrates the service-adjusted uncertainty exposure for the numerical instance. The selected feasible allocation gives H(x) = 19.399273, which lies between the minimum and maximum feasible exposure values, $H_{\min} = 17.545455$ *and* $H_{\max} = 60.363636$. The corresponding normalized exposure is $\widehat{H}(x) = 0.043296$. A lower value of $\widehat{H}(x)$ indicates lower service-adjusted uncertainty exposure relative to the feasible range.

6. Results and Discussion

The numerical results demonstrate the expected cost–service trade-off. When λ is small, greater importance is assigned to service performance; when λ increases, greater importance is assigned to robust transportation-cost minimization. The transportation allocation therefore changes with the decision-maker's preference. Increasing Γ generally raises the worst-case robust transportation cost because the uncertainty set permits a larger aggregate adverse deviation. The three baseline graphs illustrate the effects of λ and Γ on robust cost and service performance. The baseline analysis identifies the cost–service trade-off under budgeted uncertainty. The proposed extension adds H(x) to explicitly measure service-adjusted

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uncertainty exposure, and w_H provides a third decision weight alongside robust cost and service performance. The numerical calculation in Figure-4 illustrates the proposed mathematical extension.

The model can support transportation planners who must balance low operating cost against route-level service quality. The parameter λ provides a simple way to represent management preference, while Γ provides a direct control over the degree of protection against uncertain transportation costs. The numerical illustration demonstrates how the proposed exposure measure is evaluated for a feasible transportation allocation. The allocation values x_{ij} determine the transported quantities, while the corresponding service-deficiency and cost-deviation terms determine the route-level contribution to H(x). Thus, H(x) captures the combined effect of transported quantity, possible cost deviation, and service deficiency rather than considering any one factor in isolation. The normalization makes the proposed exposure criterion comparable with the other normalized criteria. The normalized criteria can then be incorporated into the weighted-sum formulation without one criterion dominating solely because of its numerical units or magnitude.

Figures 1–3 summarize the baseline behavior of robust cost and service performance under different λ and Γ values, while Figure 4 presents the newly introduced service-adjusted uncertainty exposure. Together, the figures show that the proposed model does not discard the original cost–service analysis. Instead, it adds a route-sensitive uncertainty perspective to the same decision framework. In particular, Figure4 should be read together with the route-level exposure calculation. A route contributes more to H(x) when its transported quantity, possible cost deviation, and service deficiency are simultaneously high. Thus, the proposed graph helps decision-makers move from a system-level comparison of alternatives to an interpretable identification of routes that deserve attention. A lower $\widetilde{H}$(x) indicates lower combined exposure for the same definition of the uncertainty and service measures.

The overall results indicate the practical usefulness of the proposed model. The existing robust cost–service formulation provides protection against uncertain transportation costs and represents the basic service trade-off. The proposed extension goes one step further by explicitly coupling uncertainty with service deficiency and transported quantity through H(x).

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Consequently, the decision maker receives additional information about which transportation allocations are more exposed to an undesirable combination of cost uncertainty and weak service. Therefore, the proposed model can be viewed as a more informative extension of the existing formulation rather than simply a replacement. It preserves the established transportation, service, and robust-uncertainty structure while adding a controllable third criterion through w_H. This additional criterion can support a more balanced and transparent transportation decision when cost efficiency, service quality, and uncertainty exposure all matter simultaneously.

The numerical illustration and graphical analysis demonstrate that the proposed criterion can be calculated directly from route-level data and incorporated into the existing robust optimization framework. This makes the model useful for decision situations in which a low-cost allocation is not necessarily desirable if it also produces high uncertainty exposure and service deficiency. Overall, the proposed formulation provides a clearer basis for comparing transportation alternatives because it distinguishes between ordinary cost–service performance and the additional risk created by their interaction with uncertainty.

7. Conclusion

A robust multi-objective transportation model integrating transportation cost, service performance, and budgeted uncertainty has been newly discussed in this study. The numerical experiment demonstrates baseline evidence on the cost–service trade-off and the effect of Γ. The proposed extension then newly introduces the service-adjusted uncertainty exposure measure $H(x)$ and incorporates it through the additional weight w_H in a normalized three-criterion objective. The mathematical model can therefore be useful as a decision-support framework for selecting transportation plans that are not only cost-conscious and service-oriented, but also more aware of uncertainty exposure.

7.1 Future Scope

Future studies may incorporate uncertain demand, time-dependent transportation costs, multiple transportation modes, environmental objectives, disruption scenarios, and larger computational instances.

Published by
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ISSN: 3139-6917 (Online)
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ISSN: 3139-6917 (Online)
Volume- 01, Issue-02
July 2026

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XAVERIAN QUINTESSENCE

(An International Multidisciplinary Subjects and Peer- Reviewed Journal)

Biannual Online Journal

ISSN: 3139-6917 (Online)
Volume- 01, Issue-02
July 2026

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